Int. J. Heat Mass Transfer. Vol. 12, pp. 965–979. Pergamon Press 1969. Printed in Great Britain

NONSIMILAR SOLUTIONS FOR LAMINAR FILM CONDENSATION ON A VERTICAL SURFACE

V. E. DENNY and A. F. MILLS†

Department of Engineering, University of California, Los Angeles, California 90024

(*Received* 21 *November* 1968 *and in revised form* 10 *February* 1969)

Abstract—A general computer program has been developed which solves the finite difference analogue of the conservation equations in boundary layer form for laminar film condensation. In addition, closed form analytical solutions based on the Nusselt assumptions have been extended to include the effect of a nonisothermal condenser wall. In so doing, a reference temperature for evaluating locally variable fluid properties is defined in the form $T_r = T_w + \alpha(T_s - T_w)$ and accounts for both the effects of fluid property variations and minor errors introduced by the Nusselt assumptions. In the numerical as well as the analytical developments, vapor drag was accounted for by means of an asymptotic solution of the vapor boundary layer under strong suction which is valid for most engineering situations.

This paper evaluates the validity of the extended Nusselt result in terms of the effects of (i) forced vapor flow, (ii) variable wall temperature, and (iii) variable fluid properties. Ten fluids were investigated, including water, all having Prandtl numbers greater than unity. It was found, except for severe wall temperature variations, that a value of α dependent only on fluid specie led to a less than 2 per cent discrepancy between the analytical predictions and the numerical results.

NOMENCLATURE

C_p,	heat capacity [Btu/lb °R];
Fr,	dimensionless Froude number, gl_r/v_r^2;
g,	gravitational acceleration [32·174 ft/s^2];
h,	heat transfer coefficient [Btu/ft^2 s °R];
H,	$\lambda + \int_T^{T_s} C_p \, \mathrm{d}T$ [Btu/lb];
k,	thermal conductivity [Btu/ft s °R];
l_1, l_2,	metric coefficients [ft];
l_r,	reference length (0·002 ft);
L,	length of the vertical wall (1·584 ft);
$\dot{m}$,	rate of condensation [lb/ft^2 s];
Pr,	dimensionless Prandtl number, $\mu_r C_{p_r}/k_r$;
Re,	dimensionless Reynolds number, $l_r\rho_r v_r/\mu_r$;
T,	absolute temperature [°R];
u, v,	velocity components in x, y plane [ft/s];
v_1, v_2,	velocity components in ξ_1, ξ_2 plane [ft/s];
x, y,	boundary layer coordinates [ft].

Greek symbols

α,	dimensionless parameter which defines the reference temperature $T_r = T_w + \alpha(T_s - T_w)$;
γ,	dimensionless parameter which establishes the spacing of the streamlines in the ξ_1, ξ_2 plane;
Γ,	mass flow rate of the condensate film [lb/ft s];
δ,	thickness of condensate film [ft];
ζ,	dimensionless coordinate, $l_1\xi_1/l_r$;
θ,	dimensionless temperature, $(T_s - T)/(T_s - T_w)$;
λ,	latent heat of vaporization [Btu/lb];
μ,	absolute viscosity [lb/ft s];
ν,	kinematic viscosity [ft^2/s];

---

† This work was supported in part by the State of California through the University of California Statewide Water Resources Center on Grant No. 4-442 575-20537. Computer time for the numerical calculations was provided by the Campus Computing Network of the University of California, Los Angeles.

ξ_1, ξ_2,	dimensionless coordinates related to dx and dy by $dx \equiv l_1\, d\xi_1$, $dy \equiv l_2\, d\xi_2$;
ρ,	density [lb/ft^3];
ψ,	stream function [lb/ft s];
ω,	dimensionless coordinate, $\omega \equiv \xi_2$.

Superscripts

$'$,	denotes nondimensional quantity;
t,	denotes a theoretically derived quantity.

Subscripts

e,	edge of the vapor boundary layer;
iso,	isothermal wall case;
r,	reference state;
s,	vapor–liquid interface;
v,	vapor phase;
w,	condenser wall.

Overlines

$\sim$ average with respect to x,

$$\tilde{f} = \frac{1}{x}\int_0^x f\,\mathrm{d}x;$$

$\bar{\ }$, average for $x = L, \bar{f} = \dfrac{1}{L}\displaystyle\int_0^L f\,\mathrm{d}x.$

INTRODUCTION

THE FIRST analysis of film condensation was performed by Nusselt in 1916 [1]. Since then there has been substantial further analytical work directed towards evaluating the validity of Nusselt's results and increasing the extent of the problem domain. In general, four analytical approaches have been employed. They are (i) the use of what may be called the Nusselt assumptions whereby liquid acceleration and energy convection are neglected, (ii) the use of integral forms of the boundary layer momentum and energy equations, (iii) similarity solutions of the boundary layer momentum and energy equations, (iv) perturbation methods based on the aforementioned approaches. In many cases the aspects of the specific problem investigated have been determined by the power of the analytical method rather than their relevance to the engineering problem. The present day capacity of digital computers together with recent refinements in numerical analysis allows further progress to be made. A number of restrictive assumptions can be eliminated and a broader range of situations of engineering importance investigated. Following Sparrow and Gregg [2], film condensation may be regarded as a boundary layer problem inasmuch as it is the boundary layer forms of the momentum and energy conservation equations which are to be solved. This model of the behavior of a laminar film involves only one significant assumption, that is the neglect of effects related to surface instabilities. Analyses based on this model are valid in a great many practical situations where the effects are indeed negligible. Furthermore, base solutions are provided for use in the empirical correlation of the effects of surface instabilities.

The current state of the art can be delineated by giving a brief description of the important features of the significant prior analytical work. On the assumption of negligible forced convection in the vapor phase Koh, Sparrow and Hartnett [3] obtained similarity solutions for the condensation of saturated vapor on a vertical surface. Sparrow and Gregg [4] obtained approximate similarity solutions for the condensation of saturated vapor on a horizontal tube with negligible vapor drag. Minkowycz and Sparrow [5] invoked the Nusselt assumptions to analyze condensation of water vapor on a vertical surface with superheat, air as a noncondensable gas, an interfacial resistance and variable fluid properties. The vapor flow was assumed due to liquid drag and free convection only. Koh [6] obtained similarity solutions for condensation on a horizontal plate with forced flow of a saturated vapor. Shekriladze and Gomelauri [7] used the Nusselt assumptions to analyze condensation on a plate for an arbitrary gravitational field with forced flow of saturated vapor: for a horizontal tube the effect of forced vapor flow for zero gravity was analyzed. Yang [8] used a perturbation method to study the effect of variable wall temperature for the analysis of Koh, Sparrow and Hartnett [3]. Poots and Miles [9] obtained variable property similarity solutions for water condensing on a vertical surface from quiescent saturated vapor at 212°F.

The pertinent analyses have shown that the Nusselt assumptions are valid over a remarkably large range of conditions. Taking each of the assumptions in turn, their limits of validity are as follows. Liquid acceleration effects are not negligible at high rates of condensation of low Prandtl number fluids, viz., liquid metals [2, 3, 4, 6]. The significance of this discrepancy has been very much overemphasized in the past since the heat fluxes and vapor velocities corresponding to such high condensation rates are quite unrealistic in terms of engineering or experimental practice. Koh, Sparrow and Hartnett [3] obtained solutions for the dimensionless condensation rate parameter $C_p\Delta T/\lambda$ equal to 0·01 and 0·1. It is true that values of the parameter as large as these apparently have been obtained in a number of liquid metal experimental investigations [10–14]. However, the corresponding condensation rates measured were an order of magnitude below the analytical predictions. It is now generally accepted that the true values of the temperature difference across the liquid film ΔT, were not equal to the measured $(T_s - T_w)$ where T_s is the saturation temperature corresponding to the measured vapor pressure. The discrepancy is most probably due to the presence of noncondensable gas in the vapor phase though some authors [13–15] consider the existence of a molecular-kinetic interfacial resistance a possible explanation. Energy convection effects are not negligible for the condensation of high Prandtl number liquids in reduced gravitational fields with forced vapor flow [6]. This is to be expected since in such a situation the behavior of the film approaches that of a conventional forced flow boundary layer.

In deciding on a rational approach to film condensation problems in general, the merits of the Nusselt assumptions suggest their use wherever possible. Exact solutions of the conservation equations are required to complete the picture. Since the possibilities of obtaining similar solutions have been almost exhausted, it follows that the exact solution procedure must be capable of yielding results for nonsimilar problems. In the present paper the objectives are to report the development of such a solution procedure, and its application to the study of three nonsimilar problems. The results presented are restricted to the condensation of saturated vapor of high Prandtl number liquids on a vertical surface at normal gravity; the specific problems are:

(i) The effect of a forced vapor flow parallel to the condensing surface.
(ii) The effect of longitudinal wall temperature variation.
(iii) The determination of suitable reference temperatures for use in conjunction with constant property analyses based on the Nusselt assumptions.

The exact solutions were obtained from a general computer program based on a finite difference analog to the conservation equations in boundary layer form. The complete partial differential equations were considered and thus there are no restrictions arising from similarity requirements. Furthermore, in a finite difference procedure, variable fluid properties are accounted for almost as easily as constant properties.

The problem domain was further restricted by considering a range of vapor velocities and condensation rates for which the vapor drag on the liquid film can be described by a particularly simple asymptotic shear expression. With this restriction the Nusselt assumptions yield a closed form analytical solution, allowing the results to be discussed in terms of the validity and application of the Nusselt result. It will be shown that the asymptotic shear expression is valid for most engineering applications. However, another factor is important in this context.

The removal of this restriction presents an awkward problem. A solution procedure for the coupled two phase flow problem is given in [16] and is quite straightforward. Rather, the difficulties lie in the specification of realistic boundary conditions for the vapor flow owing to the wide variety of vapor flow patterns encountered in engineering applications. Performing the computations is relatively simple, but choosing realistic cases and reporting the results is not a trivial matter. This dilemma is, of course, common to most analyses of nonsimilar boundary-layer flows. The authors' viewpoint is that the availability of the computer program, as a tool for design or research, is of greater importance than results for specific cases.

ANALYSIS AND NUMERICAL PROCEDURE

Physical model and coordinates. A schematic representation of the physical model and coordinate system is shown in Fig. 1. The coordinates are considered in terms of metric coefficients l_1 and l_2 such that $l_1\,d\xi_1$ and $l_2\,d\xi_2$ represent infinitesimal real distances. The vapor is at the saturation temperature T_s. The condenser wall is at temperature T_w, ($T_w < T_s$) which is an arbitrary function of position.

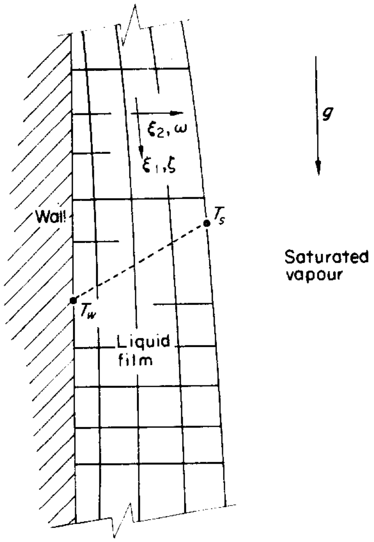

FIG. 1. Schematic diagram of the coordinate system.

Conservation equations and boundary conditions. The equations governing conservation of mass, momentum and energy for steady laminar film condensation on a plane vertical surface are respectively, as follows:

$$\frac{\partial}{\partial \xi_1}(\rho v_1 l_2) + \frac{\partial}{\partial \xi_2}(\rho v_2 l_1) = 0 \tag{1}$$

$$\frac{\rho v_1}{l_1}\frac{\partial v_1}{\partial \xi_1} + \frac{\rho v_2}{l_2}\frac{\partial v_1}{\partial \xi_2} = \frac{1}{l_1 l_2}\frac{\partial}{\partial \xi_2}\left(\frac{l_1 \mu}{l_2}\frac{\partial v_1}{\partial \xi_2}\right) + g(\rho - \rho_v) \tag{2}$$

$$\frac{\rho C_p v_1}{l_1}\frac{\partial T}{\partial \xi_1} + \frac{\rho C_p v_2}{l_2}\frac{\partial T}{\partial \xi_2} = \frac{1}{l_1 l_2}\frac{\partial}{\partial \xi_2}\left(\frac{l_1 k}{l_2}\frac{\partial T}{\partial \xi_2}\right). \tag{3}$$

Viscous dissipation and compressibility effects are omitted from the energy equation since low velocity liquid flow is under consideration. At the condenser wall $\xi_2 = 0$ and the boundary conditions are:

$$v_1 = 0;\ v_2 = 0;\ T = T_w(\xi_1). \tag{4}$$

At the liquid surface there is the requirement of continuity of momentum flux between liquid and vapor phases. For a boundary-layer flow under strong suction, a simple asymptotic solution of the momentum conservation equation yields the required boundary condition [16]. In this limit the local shear stress is identically equal to the momentum given up by the condensing vapor $(\rho_v v_{v,s}(u_{v,e} - v_1))$. Thus, since

$$-\rho_v v_{v,s} \equiv \frac{1}{l_1}\frac{d\psi_s}{d\xi_1},$$

the conditions at the liquid surface are:

$$\frac{\mu}{l_2}\frac{\partial v_1}{\partial \xi_2} = \frac{1}{l_1}\frac{d\psi_s}{d\xi_1}(u_{v,e} - v_1);\ T = T_s. \tag{5}$$

The local thickness of the condensate film is determined from an overall enthalpy balance on a length of film between a reference location $\xi_{1,0}$ and the location in question, ξ_1:

$$\int_0^\delta \rho v_1 H l_2 \,\mathrm{d}\xi_2 \Big|_{\xi_1} = \int_{\xi_{1,0}}^{\xi_1} \frac{k}{l_2}\frac{\partial T}{\partial \xi_2} l_1 \,\mathrm{d}\xi_1 \Big|_{\xi_2 = 0} + \int_0^\delta \rho v_1 H l_2 \,\mathrm{d}\xi_2 \Big|_{\xi_{1,0}} \qquad (6)$$

Modifying a technique developed by Patankar and Spalding [17] the cross-stream variable ξ_2 is defined in terms of the stream function as:

$$\xi_2 = \left(\frac{\psi - \psi_w}{\psi_s - \psi_w}\right)^{1/\gamma} = \omega \qquad (7)$$

where a value of 2 for the constant γ proved to minimize the truncation errors in the computational procedure. Hence:

$$\psi = \psi_w + \Delta\psi\omega^\gamma, \text{ where } \Delta\psi = \psi_s - \psi_w \qquad (8)$$

and

$$l_2 = \frac{\gamma\Delta\psi\omega^{\gamma-1}}{\rho v_1}, \; \rho v_2 = -\frac{1}{l_1}\left[\frac{\mathrm{d}\psi_w}{\mathrm{d}\xi_1} + \frac{\mathrm{d}\Delta\psi}{\mathrm{d}\xi_1}\omega^\gamma\right]. \qquad (9)$$

Introducing the above relations into Equations (2) and (3), performing the indicated differentiation and rendering the final forms dimensionless yields:

$$\frac{\partial v_1'}{\partial \zeta} - \frac{1}{\gamma\Delta\psi'\omega^{\gamma-1}}\left(\frac{\mathrm{d}\psi_w'}{\mathrm{d}\zeta} + \frac{\mathrm{d}\Delta\psi'}{\mathrm{d}\zeta}\omega^\gamma\right)\frac{\partial v_1'}{\partial \omega} = \frac{1}{\omega^{\gamma-1}}\frac{\partial}{\partial\omega}\left(\frac{\rho'\mu' l_1' v_1'}{\gamma^2 Re(\Delta\psi')^2\,\omega^{\gamma-1}}\frac{\partial v_1'}{\partial\omega}\right) + \frac{l_1'}{\rho' v_1'} Fr(\rho' - \rho_v') \qquad (10)$$

$$\frac{\partial \theta}{\partial \zeta} - \frac{1}{\gamma\Delta\psi'\omega^{\gamma-1}}\left(\frac{\mathrm{d}\psi_w'}{\mathrm{d}\zeta} + \frac{\mathrm{d}\Delta\psi'}{\mathrm{d}\zeta}\omega^\gamma\right)\frac{\partial \theta}{\partial \omega} = \frac{1}{C_p'\omega^{\gamma-1}}\frac{\partial}{\partial\omega}\left(\frac{\rho' k' l_1' v_1'}{\gamma^2 RePr(\Delta\psi')^2\,\omega^{\gamma-1}}\frac{\partial \theta}{\partial\omega}\right). \qquad (11)$$

The boundary conditions, Equations (4) and (5) transform into:

$$\omega = 0 : v_1' = 0; \frac{\mathrm{d}\psi_w'}{\mathrm{d}\zeta} = 0; \theta = \theta_w \qquad (12)$$

$$\omega = 1 : \frac{\partial v_1'}{\partial\omega} = \frac{\gamma Re}{l_1'\mu'\rho'}\Delta\psi'\frac{\mathrm{d}\psi_s'}{\mathrm{d}\zeta}\left(\frac{u_{v,e}'}{v_1'} - 1\right); \theta = 0. \qquad (13)$$

Finally the overall enthalpy balance, Equation (6), transforms into:

$$\gamma\Delta\psi'\int_0^1 \omega^{\gamma-1}\frac{H}{C_{p_r}\overline{\Delta T}}\,\mathrm{d}\omega\Big|_\zeta = -\frac{1}{RePr}\int_{\zeta_0}^{\zeta}\frac{\rho' k' v_1' l_1'}{\gamma\Delta\psi'\omega^{\gamma-1}}\frac{\partial\theta}{\partial\omega}\,\mathrm{d}\zeta\Big|_{\omega=0} + \gamma\Delta\psi'\int_0^1 \omega^{\gamma-1}\frac{H}{C_{p_r}\overline{\Delta T}}\,\mathrm{d}\omega\Big|_{\zeta_0}. \qquad (14)$$

Numerical procedure. Equations (10) and (11) are of the general form:

$$\frac{\partial\Omega}{\partial\zeta} + A\frac{\partial\Omega}{\partial\omega} = B\frac{\partial}{\partial\omega}\left(C\frac{\partial\Omega}{\partial\omega}\right) + D. \qquad (15)$$

Difference analogues to these equations are obtained by evaluating the left-hand sides as integral averages over a computational cell bounded by $\zeta - \Delta\zeta : \zeta$ and $\omega - \Delta\omega/2 : \omega + \Delta\omega/2$, while evaluating the right-hand sides at the node points $\zeta : (\omega - \Delta\omega, \omega, \omega + \Delta\omega)$. Such a formulation involving node points $\zeta : (\omega - \Delta\omega, \omega, \omega + \Delta\omega)$ and $\zeta - \Delta\zeta : (\omega - \Delta\omega, \omega, \omega + \Delta\omega)$ has been found to be unconditionally stable [17]. This procedure leads to coupled sets of nonlinear algebraic equations of the form:

$$E\Omega(\zeta, \omega - \Delta\omega) + F\Omega(\zeta, \omega) + G\Omega(\zeta, \omega + \Delta\omega) = H \qquad (16)$$

where the position dependent functionals E through H involve the dependent variables at the node set $\zeta : (\omega - \Delta\omega, \omega, \omega + \Delta\omega)$. Iterative procedures are used to advance the solution from $\zeta - \Delta\zeta$ to ζ. With the exception of the shear boundary condition at $\omega = 1$, the boundary conditions are of the Dirichlet type. The analogue of the shear condition is obtained via a central difference form for $\partial v'_1 | \partial\omega$ centered on $\omega = 1$. Thus the set of algebraic equations for v'_1 is one member greater than that for θ. The overall numerical solution is closed by satisfying the enthalpy balance, equation (14). Specifically the solution is advanced from one step to the next by performing the following operations (i) guess $d\psi'_s | d\zeta$, (ii) approximate the velocity and temperature distributions, (iii) extract an improved value of $d\psi'_s | d\zeta$ from the enthalpy balance, (iv) repeat operations (ii) and (iii) until the desired convergence is obtained. In order to start the numerical solution, initial velocity and temperature profiles are obtained from an appropriate analysis based on the Nusselt assumptions and transformed into the (ζ, ω) coordinate system. Numerical stability for the initial steps is ensured by using a weighted average of the analytical and numerical solutions. The weighting procedure allows the numerical solution to take over after a predetermined number of steps. The reliability of the numerical method has been established by reproducing the constant property similar solutions of Sparrow and Gregg [2] and Koh, Sparrow and Hartnett [3]. The results are shown in Fig. 2. The computational procedures are rapid and reliable. Typical machine times (IBM 360|75 system) to compute the heat transfer for condensation on a vertical plate one foot long are in the range 10–30 sec.

A listing of the computer program together with instructions for its use may be found in [16]. The program contains a number of additional options which allow a wide range of film

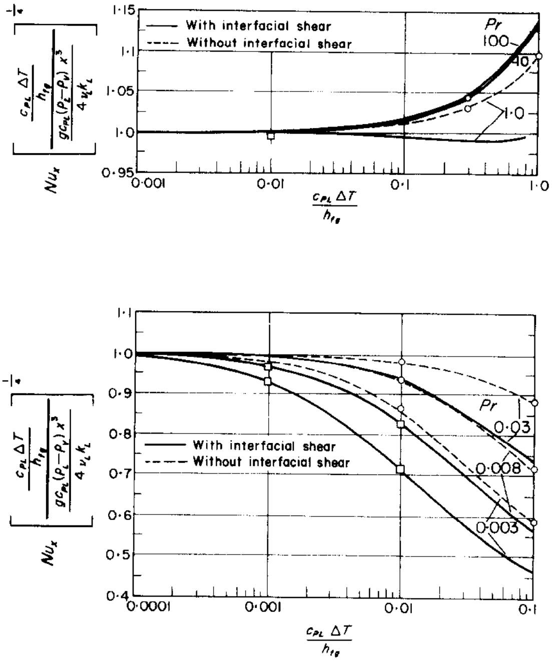

FIG. 2. Comparison of numerical solutions with similar solutions [2, 3] at zero vapor velocity. The numerical solutions with and without interfacial shear are represented by the symbols □ and ○ respectively.

Table 1. References for the thermophysical properties

Liquid	Density (ρ)	Specific heat (C_p)	Thermal conductivity (k)	Latent heat (λ)	Viscosity (μ)
Water	5, 27	27, 32	23, 27	27	5, 31
Ethyl alcohol	25	23	23	25	25
n-propyl alcohol	18	18	33	38	18, 36
n-butyl alcohol	18	19	28	18, 29	28
t-butyl alcohol	18, 36	18	33	34, 37, 38	18, 36
Ethylene glycol	18	18, 23	33	19	18, 19
Glycerol	18	20	33	19	18, 20
Carbon tetrachloride	18	23	23	19	19
Propane	26	23	22	26	24
Ammonia	19	20	23	19	20

condensation problems to be analyzed. These include, for example, condensation on a horizontal cylinder, wall suction and the magnetohydrodynamic problem for liquid metals.

Thermophysical properties. For each of the fluids investigated the available thermophysical property data was surveyed and an attempt made to identify the most reliable sources. Table 1 presents the references of the sources chosen for each of the properties required. The data was expressed in terms of accurate algebraic expressions by a least mean squares procedure. For density, specific heat, thermal conductivity and latent heat, cubic polynomials in temperature were used while for viscosity, inverse powers of temperature proved more suitable.

Analysis based on Nusselt assumption. In order that a more general assessment of the merits of the Nusselt assumptions could be made, a prior analysis of Shekriladze and Gomelauri [7] has been extended to include the effects of variable properties and longitudinal variation of wall temperature. Details of the analysis, as well as the additional assumptions required, are presented in Appendix A.

RESULTS AND DISCUSSION

Computational errors. Within the assumptions made in reducing the conservation equations to boundary layer form, errors in the numerical results stem from discretization errors, incomplete convergence of iterative procedures, arbitrariness of the initial conditions, and uncertainties in the thermophysical property data. Since the Nusselt assumptions (small effect of inertial and convective terms) are uniformly excellent for the fluids studied, the heat transfer results may be conveniently reported in terms of equations (A.11) and (A.13), using the reference temperature concept to handle variable properties. Thus errors in the property data need not be discussed since improvements in their accuracy can automatically be accounted for through the analytical expressions.

The effect of discretization errors was explored by studying in detail a typical case (CCl_4, $\Delta T = 20\,^{\circ}R$, $u_{v,e} = 20$ ft/s). Numerical experiments were performed to determine the effect of $\Delta\omega$ (holding $\Delta\zeta$ constant) as well as $\Delta\zeta$ (holding $\Delta\omega$ constant) on the heat transfer results. Successive halvings of $\Delta\omega$ from 0·1 to 0·0125 changed the heat transfer results by less than 0·25 per cent, while halving $\Delta\zeta$ from 3·98 to 1·99 resulted in an 0·1 per cent change. Since problems arose in the accurate manipulation of numbers as $\Delta\omega$ became small, it was necessary to establish convergence criteria for the iterative procedures which would ensure that the dependent variables were correct to six significant figures. For larger values of $\Delta\omega$ (0·05 and 0·1), it was found that five significant figures were sufficient. Finally, since the mathematical problem is initial value in nature, uncertainties in the initial conditions would be expected to propagate (with diminished amplitude) as the solution is advanced. It was found that artificial perturbations in the initial conditions of about 6·4 per cent were diminished to less than 0·1 per cent upon reaching the end of the plate ($\xi_1 = 1{\cdot}584$ ft). Since the errors introduced by the analytical solution used to construct the initial conditions were never greater than one per cent, such errors proved negligible. In conclusion, the authors are confident that the overall numerical results, which were obtained using $\Delta\zeta = 3{\cdot}98$ and $\Delta\omega = 0{\cdot}05$, are in error by less than 0·5 per cent.

Isothermal wall. The numerical solutions of the system of equations (10)–(14) were obtained at various saturation temperatures for the ten fluids listed in Table 1. Velocity and temperature distributions, local and average heat transfer coefficients, and local condensation and mass flow rates were obtained along a plate 1·584 ft long at ΔT's and $u_{v,e}$'s in the ranges 3–50°F and 0–200 ft/s respectively. The effect of vapor drag on the local heat transfer coefficient is shown in Fig. 3. The film Reynolds number, $4\Gamma/\mu$, at the end of the plate usually exceeded 1800, thereby insuring that any conclusions would be valid for all values of Reynolds number in the laminar regime. Only the heat transfer results will be reported here. Since the reference temperature

T_r is defined as $T_r \equiv T_w + \alpha\Delta T$, it is convenient to do this in terms of equation (A.11) by reporting the thermophysical property sources, T_s, ΔT, $u_{v,e}$, and, of course, α. Clearly, this approach does not account for the effect of variable properties alone since inertial and convective effects (though small) were neglected in deriving the analytical expressions. Due to the fact that α proved to be a weak function of all parameters except fluid specie, representative values could be selected such that the maximum error in results calculated using (A.11) would be less than 2 per cent. A summary of the results appears in Table 2.

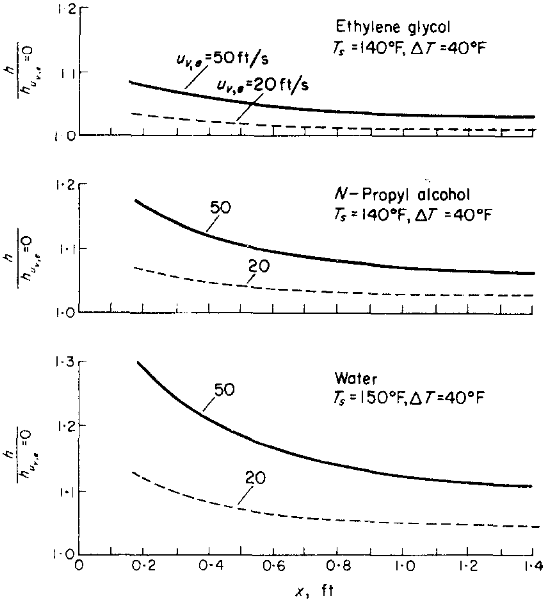

FIG. 3. Effect of vapor drag on the local heat transfer coefficient for ethylene glycol, n-propyl alcohol and water.

Attempts were made to correlate α in terms of the various parameters of the problem (such as, for example, $u_{v,e}$, Pr and $\lambda/C_P\Delta T$), however, no useful result has been obtained. For the present, it has been observed that, for the most part, α increases with increasing $u_{v,e}$ and decreases with decreasing $dh^l_{iso}|dT_r$. Exceptions to the latter were observed for propane and ammonia, for which $dh^l_{iso}|dT_r$ in contrast with the other fluids, is negative. In addition, as would be expected, the ratio $h|\bar{h}$ equalled 4/3 when $u_{v,e}$ equalled zero and approached asymptotically the limiting value of two as $u_{v,e}$ became large. Finally, it is observed that the value for water (0·33) agrees favorably with the value of 0·31 previously reported by Minkowycz and Sparrow [5] and the range of values (0·26–0·33) reported by Poots and Miles [9].

Non-isothermal wall. The effects of variable wall temperature on condensation heat transfer were studied for water only. Both co-current as well as counter-current flow of coolant were considered, where for co-current flow the temperature drop across the film was chosen to be of the form

$$\Delta T = \Delta T_0 - Mx^m + Nx^n$$

while for counter-current flow it was

$$\Delta T = \Delta T_0 + Mx^m$$

In addition, $\Delta T(x)$ was constructed such that

Table 2. Reference parameters for computing heat transfer coefficients using equation (A.11)

Fluid	T_s, deg F	ΔT, deg F	$u_{v,e}$, ft/sec	α
Carbon tetrachloride	60, 90, 120	10, 20, 40	20, 200	0·07
Ethyl alcohol	80, 120, 160	10, 20, 40	0, 20, 50, 200	0·12
n-propyl alcohol	140	20, 40	0, 20, 50	0·15
n-butyl alcohol	70, 100, 130	40	20, 200	0·25
t-butyl alcohol	140	20, 40	0, 20, 50	0·29
Ethylene glycol	140	40	0, 20, 50	0·29
Glycerol	140	40	0, 20, 50	0·32
Water	50, 80, 110, 150, 212	3, 10, 20, 40, 50	0, 20, 50, 200	0·33
Ammonia	30, 80, 120	40	20, 200	0·61
Propane	160, 180, 200	40	20, 200	1·00

the average temperature drop, $\overline{\Delta T}$, for the overall plate was 20°F.

Comparisons of the local heat transfer coefficient, as predicted by equation (A.11), with values calculated from the numerical solution of the full boundary-layer equations appear in Fig. 4 in terms of two fixed values of the reference parameter α, 0·400 for the counter-current flow cases and 0·345 for the co-current cases. In a strict sense, a single fixed value of α would not be expected to represent the data precisely. However, if $\alpha = \frac{1}{3}$ were assigned to any of the cases shown in Fig. 4, the maximum discrepancy between the analytical and numerical results would be less than 3 per cent. Since $\alpha = \frac{1}{3}$ is also the value recommended for the isothermal wall cases, it should be accepted as the best value for water.

Since the expression for h^t is somewhat cumbersome, it is of interest to explore an alternative in which h^t is calculated through the ratio $h^t | h^t_{iso}$ as discussed in Appendix A. Comparisons of $h^t | h^t_{iso}$ with values extracted from the numerical solutions appear in columns 13 and 14 of Table 3. The coefficient h^t_{iso} is computed as if the local value of ΔT prevailed from the leading edge; it has no physical significance but does allow a convenient comparison between the theoretical and numerical solutions. As can be seen, the percentage differences are little greater than the expected errors in the numerical results themselves. Thus, it would seem appropriate to calculate h^t through equations (A.11) and (A.13), assuming the wall to be isothermal when applying (A.11). Although it may possibly be useful to develop similar comparisons for $\bar{h}^t | \bar{h}^t_{iso}$ with the corresponding numerical results, this has not been done because the integral forms which result are not tractable, even when $u_{v,e} = 0$, due to the dependence of the co-

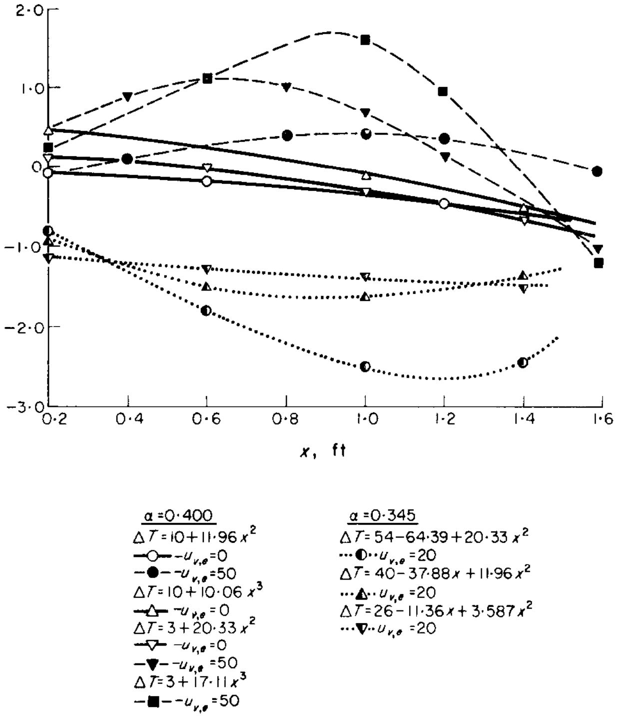

FIG. 4. Comparisons of equation (A.11) with the numerical results tor condensation of water on a vertical nonisothermal wall.

Table 3. Laminar film condensation of water on a non-isothermal wall ($\overline{\Delta T} = 20^{\circ} F$)

T_s	ΔT_0	ΔT_L	M	m	N	n	$u_{v,e}$	$\bar{h}/\bar{h}_{iso}$	x	ΔT	h	h/h_{iso}	h^t/h^t_{iso}
Co-current coolant flow													
110	26	17	11·36	1	3·59	2	20	0·997	0·669	20	0·2570	0·957	0·962
110	40	10	37·88	1	11·96	2	0	0·987	0·669	20	0·2346	0·905	0·909
110	40	10	37·88	1	11·96	2	20	1·002	0·669	20	0·2396	0·892	0·895
110	54	3	64·39	1	20·33	2	20	0·999	0·669	20	0·2266	0·844	0·846
212	30	10	12·63	1			8	0·990	0·792	20	0·2985	0·939	0·946
212	30	10	12·63	1			50	1·022	0·792	20	0·3240	0·898	0·900
212	23	17	3·79	1			50	0·997	0·792	20	0·3457	0·959	0·972
Counter-current coolant flow													
110	3	54	20·33	2			0	0·981	1·349	40	0·2203	1·273	1·271
									0·915	20	0·2973	1·237	1·232
110	3	54	20·33	2			20	0·986	1·349	40	0·2373	1·328	1·339
									0·915	20	0·3164	1·280	1·282
110	3	54	20·33	2			50	1·000	1·349	40	0·2649	1·414	1·431
									0·915	20	0·3493	1·350	1·355
110	10	40	11·96	2			0	1·000	1·584	40	0·1984	1·189	1·189
110	10	40	11·96	2			20	0·996	1·584	40	0·2101	1·221	1·228
110	10	40	11·96	2			50	0·995	1·584	40	0·2288	1·277	1·285
110	3	71	17·11	3			0	0·965	1·294	40	0·2355	1·345	1·344
110	3	71	17·11	3			20	0·977	1·294	40	0·2569	1·422	1·425
110	3	71	17·11	3			50	1·003	1·294	40	0·2923	1·542	1·561
110	10	50	10·06	3			0	0·995	1·439	40	0·2102	1·232	1·231
110	10	50	10·06	3			20	0·992	1·439	40	0·2244	1·279	1·289
110	10	50	10·06	3			50	0·995	1·439	40	0·2477	1·348	1·360
212	3	37	21·46	1			0	1·003	0·792	20	0·3673	1·152	1·149
212	10	30	12·63	1			20	0·998	0·792	20	0·4134	1·145	1·139
212	17	23	3·79	1			50	0·999	0·792	20	0·3762	1·042	1·037

efficients f_1 and f_2 on position. However, it is instructive to consider a convenient limiting case in which the product $f_1^2 f_2$ is arbitrarily set equal to 1. Thus, for $u_{v,e} = 0$ and $f_1^2 f_2 = 1$, it may easily be shown that:

$$\frac{\bar{h}}{\bar{h}^t_{iso}} = 1 - \left(1 - \frac{\Delta T_0}{\overline{\Delta T}}\right)\left(\frac{m}{4m + 3}\right)$$

for counter-current coolant flow. Here, $\bar{h}^t_{iso}$ is the average heat transfer coefficient for an isothermal wall problem for which $\Delta T_{iso} = \Delta T_0 + ML^m/(m + 1)$. We note that $\bar{h}/\bar{h}^t_{iso}$ decreases with increasing m and increases with increasing ΔT_0, which is in line with the trends exhibited by $\bar{h}/\bar{h}_{iso}$ listed in column 9 of Table 3.

Significance of the Results. The use of a reference temperature concept to account for the omission of the acceleration and convective terms as well as variable fluid properties, facilitates the use of constant property analyses based on the Nusselt assumptions. The values of the parameter α established in this investigation allow the closed form analytical expressions [equations (A.11) and (A.13)] to be used over a wide range of the pertinent parameters. An evaluation of the validity of these results in relation to engineering problems requires the discussion of a number of issues.

(i) For water condensing on a vertical surface waves appear at a film Reynolds number ($4\Gamma/\mu$) of about 30 and steadily increase in intensity with increasing Reynolds number. Experimental results of Zazuli reported by Kutateladze [39] indicate a steady increase in heat transfer above the Nusselt value as the Reynolds number increases from 30 to 100. At $Re = 100$ a 13·8 per cent increase was observed. A more marked increase in heat transfer occurs at Reynolds numbers between 300 and 2000 and is usually attributed to a conventional transition to turbulent flow [40]. It appears from these observations that deviations from analyses assuming laminar flow and an absence of surface instability, will occur first due to an onset of surface waves. For liquids other than water, data is too limited to yield quantitative criteria for the onset of waves. A review of the literature on this topic is given by Fulford [41] to which the interested reader is referred. The results of the present investigation are considered to be valid for situations where surface waves are not observed. In addition the results serve as base solutions for the empirical correlation of the effects of waves and turbulence; such use is necessitated by the absence of reliable analyses of these effects.

(ii) The reference temperature parameter α is sufficiently insensitive to inaccuracies in the physical property data to remain unchanged within the stated accuracy of the heat transfer results. Thus it is recommended that improved property data, as it becomes available, be used directly.

(iii) The wall temperature variations considered included cases more severe than those encountered in general practice. Thus the utility of the analytical solution has been established and it may be applied to specific problems of interest.

(iv) The rationale underlying the use of the asymptotic shear expression was given in the introduction. It remains to place a lower limit on the condensation rate for which the corresponding suction velocity is large enough to insure the validity of the closed form analytical results. The accuracy of the asymptotic shear expression has been investigated in Reference {16] by obtaining numerical solutions from a finite difference analogue to the momentum conservation equation governing the vapor phase flow. For the range of variables considered in the present investigation the conclusion is that the error in the heat transfer introduced by the asymptotic shear expression is less than 1 per cent for

$$\frac{v_{v,s}}{u_{v,e}}\sqrt{\frac{x u_{v,e}}{\nu_v}} > 2.$$

Precision with respect to this criterion is not of great importance since the parameters $v_{v,s}$, $u_{v,e}$, ν_v and x vary over many orders of magnitude. Table 4 presents data typical of engineering applications for the various fluids. For condensation conditions other than those specified, the values given may be used as base figures for additional calculations. It is of interest to note that the asymptotic shear expression can be seen to be valid for the $u_{v,e} = 0$ situation by examining the results of Koh, Sparrow and Hartnett [3]. In that analysis the parameter $(\rho\mu/\rho_v\mu_v)$ characterized the vapor drag; however their table of results shows that, for a given

Table 4. Values of $\frac{v_{v,s}}{u_{v,e}}\sqrt{\left(\frac{x u_{v,e}}{\nu}\right)}$ *appropriate to film condensation situations*

Fluid	Saturation temperature (°F)	Temperature difference (°F)	Vapor velocity, $u_{v,e}$ (ft/s)	Location (ft)	$\frac{v_s}{u_e}\sqrt{\left(\frac{x u_{v,e}}{\nu}\right)}$
Water	110	20	20	1	5·98
Ethyl alcohol	120	20	20	1	1·56
n-propyl alcohol	140	20	20	1	18·7
n-butyl alcohol	110	40	20	1	10·4
t-butyl alcohol	140	20	20	1	0·90
Ethylene glycol	140	40	20	1	19·5
Glycerol	140	40	20	1	108·0
Carbon Tetrachloride	90	20	20	1	2·21
Propane	180	40	20	1	0·54
Ammonia	40	40	20	1	7·54

condensation rate, the heat transfer coefficient is independent of this parameter.

The manner in which the reference temperature concept has been established in this investigation has a broader significance than the direct use of the analytical solutions. Of particular importance is its utility in performing analyses of the many important film condensation problems which require consideration of a coupled two-phase flow. The various nonsimilar noncondensable gas problems are good examples of such problems. It is suggested that the liquid phase in such a situation may be analyzed with constant properties and the Nusselt assumptions, greatly simplifying the overall analysis. The reference temperature parameter α would not be significantly affected by the slightly different vapor drag and nonsimilarity effects.

REFERENCES

1. W. Nusselt, Die Oberflachen-Kondensation des Wasserdampfes, *Z. Ver. D-Ing.* **60**, 541–546 (1916).
2. E. M. Sparrow and J. L. Gregg, A boundary-layer treatment of laminar film condensation, *Trans. Am. Soc. mech. Engrs, Ser. C*, **81**, 13–18 (1959).
3. J. C. Y. Koh, E. M. Sparrow and J. P. Hartnett, Two-phase boundary-layer in laminar film condensation, *Int. J. Heat Mass Transfer* **2**, 69–82 (1961).
4. E. M. Sparrow and J. L. Gregg, Laminar condensation heat transfer on a horizontal cylinder, *Trans. Am. Soc. mech. Engrs. Ser. C*, **81**, 291–296 (1959).
5. W. J. Minkowycz and E. M. Sparrow, Condensation heat transfer in the presence of noncondensables, interfacial resistance, superheating, variable properties, and diffusion, *Int. J. Heat Mass Transfer* **9**, 1125–1144 (1966).
6. J. C. Y. Koh, Film condensation in a forced-convection boundary-layer flow, *Int. J. Heat Mass Transfer* **5**, 941–953 (1962).
7. I. G. Shekriladze and V. I. Gomelauri, Theoretical study of laminar film condensation of a flowing vapor, *Int. J. Heat Mass Transfer* **9**, 581–591 (1966).
8. K. T. Yang, Laminar film condensation on a vertical nonisothermal plate, *Trans. Am. Soc. mech. Engrs. Ser.* E, **88**, 203–205 (1966).
9. G. Poots and R. G. Miles, Effects of variable physical properties on laminar film condensation of saturated steam on a vertical flat plate, *Int. J. Heat Mass Transfer* **10**, 1677–1692 (1967).
10. B. Misra and C. F. Bonilla, Heat transfer in the condensation of metal vapors: mercury and sodium up to atmospheric pressure, *Chem. Engng Prog. Symp. Ser.* **52**, 7–21 (1956).
11. S. P. Sukhatme and W. M. Rohsenow, Heat transfer during film condensation of a liquid metal vapor, *Trans. Am. Soc. mech. Engrs. Ser. C*, **88**, 19–28 (1966).
12. R. E. Barry and R. E. Balzhiser, Condensation of sodium at high heat fluxes, *Proc. 3rd International Heat Transfer Conference, Chicago*, Vol. 2, pp. 318–328. A.I.Ch.E. (1966).
13. D. A. Labuntsov and S. I. Smirnov, Heat transfer in condensation of liquid metal vapors, *Proc. 3rd International Heat Transfer Conference, Chicago*, Vol. 2, pp. 329–336, A.I.Ch.E. (1966).
14. D. G. Kroger and W. M. Rohsenow, Film condensation of saturated potassium vapor, *Int. J. Heat Mass Transfer* **10**, 1891–1894 (1967).
15. I. T. Aladyev, N. S. Kondratyev, V. A. Mukhin, M. E. Kipshidze, Inna Parfentyeva and V. V. Kisselev, Thermal resistance of phase transition with condensation of potassium vapor, *Proc. 3rd International Heat Transfer Conference, Chicago*, Vol. 2, pp. 313–317, A.I.Ch.E. (1966).
16. V. E. Denny, A. F. Mills, J. Gardiner and Y. Knobel, A general computer program for the solution of the equations governing laminar film condensation, Report No. 69–70, Department of Engineering, University of California, Los Angeles, 1969.
17. S. V. Patankar and D. B. Spalding, A finite difference procedure for solving the equations of the two-dimensional boundary layer, *Int. J. Heat Mass Transfer* **10**, 1389–1411 (1967).
18. J. Timmermans, *Physico-Chemical Constants for Pure Organic Compounds*, Elsevier, Amsterdam (1950).
19. C. D. Hodgman, *Handbook of Chemistry and Physics*, 42nd Edn, Chemical Rubber Pub. Co. (1960–61).
20. F. Kreith, *Principles of Heat Transfer*, 2nd Edn, pp. 595–599, International Textbook Co., Pennsylvania (1966).
21. *American Society of Refrigeration Data Book, Design Volume*, 9th Edn, pp. 7–07 (1955).
22. D. E. Leng and E. W. Cummings, Thermal conductivity of propane *Ind. Engng Chem.* **49**, 2042–2045 (1957).
23. *Thermophysical Properties Research Center Data Book*, Vol. 2, Purdue University, Indiana (1966).
24. G. W. Swift, J. A. Christy and F. Kurata, Liquid viscosities of methane and propane, *A.I.Ch.E. Jl*, **5**, 98–102 (1959).
25. *Ethyl Alcohol*, Enjay Chemical Pub. Co., New York (1962).
26. W. V. Stearns and E. J. George, Thermodynamic properties of propane, *Ind. Engng Chem.* **35**, 602–607 (1943).
27. *Steam Tables*, Combustion Engineering Inc., New York (1940).
28. W. H. McAdams, *Heat Transmission*, 3rd Edn, McGraw-Hill, New York (1954).
29. *International Critical Tables*, Vol. 5, McGraw-Hill, New York (1928).
30. R. R. Dreisbach, *Vapor Pressure-Temperature Data for Organic Compounds*, The Dow Chemical Co., Midland, Michigan (1946).
31. E. A. Bruges, B. Latto and A. K. Ray, New corrections and tables of the coefficient of viscosity of water and

steam up to 1000 bar and 1000°C, *Int. J. Heat Mass Transfer* **9**, 465–480 (1966).

32. A. E. Sheindlin, E. E. Shpil'rain and V. V. Sychev, Teploernkost' C_p Vodi i Vodianogo para na Linii na Nasyshcheniia, *Teploenergetika*, Issue No. 7, 23–27 (1960).
33. B. C. Sakiadis and J. Coates, Studies of thermal conductivity of liquids, *A.I.Ch.E. Jl* **1**, 275–288 (1955).
34. L. H. Krone and R. G. Johnson, Thermodynamic properties of tertiary butyl alcohol, *A.I.Ch.E. Jl* **2**, 552–554 (1956).
35. W. J. Scheffy and E. F. Johnson, Thermal conductivities of liquids at high temperatures, *J. Chem. Engng Data* **6**, 245–249 (1961).
36. P. Bamelis, P. Huyskens and E. Meeussen, Influence de L'Association des Alcools sur la Viscosite des Solutions, *J. Chim. Phys.* **62**, 158–170 (1965).
37. K. C. McCurdy and K. J. Laidler, Heats of vaporization of a series of aliphatic alcohols, *Can. J. Chem.* **41**, 1867–1871 (1963).
38. E. T. Beynon and J. J. McKetta, The thermodynamic properties of 2-methyl-2-propanol, *J. Phys. Chem.* **67**, 2761–2765 (1963).
39. S. S. Kutateladze, *Fundamentals of Heat Transfer*, pp. 298–326, Edward Arnold, London (1963).
40. W. M. Rohsenow, J. H. Weber and A. J. Ling, Effect of vapor velocity on laminar and turbulent film condensation, *Trans. Am. Soc. mech. Engrs* **78**, 1637, 1643 (1956).
41. G. D. Fulford, The flow of liquids in thin films, in *Advances in Chemical Engineering*, Vol. 5, Academic Press, New York (1964).

APPENDIX A

Analysis Based on Nusselt Assumptions

The objective of this analysis is to develop closed form analytical solutions for laminar film condensation of a pure saturated vapor on a vertical non-isothermal wall. The analysis includes the effects of variable fluid properties and of vapor drag. The effect of temperature on the fluid properties is treated by means of the reference temperature concept. Vapor drag is accounted for by means of an asymptotic expression for shear deduced from an analysis of boundary layer flows subject to strong suction [16]. The analysis is restricted to situations where the Nusselt assumptions are valid, this restriction being of little consequence in terms of the engineering problem. The boundary value problem to be considered is:

$$\frac{d^2u}{dy^2} = -\frac{g}{\nu}\left(1 - \frac{\rho_v}{\rho}\right) = -\frac{g^*}{\nu}$$

$$\frac{d^2T}{dy^2} = 0$$

$$T = T_w(x) \text{ and } u = 0 \text{ at } y = 0$$

$$T = T_s \text{ and } \mu\frac{du}{dy} = \dot{m}(u_{v,e} - u) \text{ at } y = \delta$$

where

$$\dot{m} = -\rho_v v_{v,s}.$$

It may readily be demonstrated that the velocity and temperature distributions consistent with the above equations are:

$$T_s - T = (T_s - T_w)(1 - y/\delta) = \Delta T(1 - y/\delta) \tag{A.1}$$

$$u = [(1 - 0\cdot 5y/\delta)(g^*\delta^2/\nu) + (P)(u_{v,e} - 0\cdot 5g^*\delta^2/\nu)]\,[y/\delta] \tag{A.2}$$

where

$$P = (\dot{m}\delta/\mu)/(1 + \dot{m}\delta/\mu).$$

The local film thickness δ, as well as the condensation rate $\dot{m}$, are obtained from the overall enthalpy and mass balances

$$\int_0^x k\frac{dT}{dy}\,dx = \int_0^\delta \rho u[\lambda + C_p(T_s - T)]\,dy \tag{A.3}$$

$$\Gamma = \int_0^\delta \rho u\,dy. \tag{A.4}$$

Thus, if (A.1) and (A.2) are substituted in (A.3), there results after considerable algebra

$$\int_0^x \frac{k\Delta T}{\delta}\,dx = a\delta + b\delta^3 \tag{A.5}$$

where

$$a = (0\cdot 5\,\rho u_{v,e}P)(\lambda + 0\cdot 354\,C_p\Delta T)$$

and

$$b = (\rho g^*/3\nu)(1 - 0\cdot 75\,P)(\lambda + 0\cdot 354\,C_p\Delta T).$$

(The various coefficients of $C_p\Delta T$ which evolve naturally in the algebraic manipulations, 1/3 and 3/8, have been replaced by their average value, 0·354, to effect a convenient simplification in the coefficients a and b).

If the nonlinear integral equation (A.5) is now differentiated, the result may be rearranged to obtain

$$k\Delta T\,\mathrm{d}(x) = \mathrm{d}(a\delta^2 + b\delta^4) - 0{\cdot}5a\,\mathrm{d}(\delta^2) - 0{\cdot}25b\,\mathrm{d}(\delta^4)$$

which may then be integrated to yield

$$k\widetilde{\Delta T}x = (a - 0{\cdot}5\tilde{a})\,\delta^2 + (b - 0{\cdot}25\tilde{b})\,\delta^4$$

(The quantities $k\widetilde{\Delta T}$, $\tilde{a}$, and $\tilde{b}$ are integral mean values over the interval 0, x). The film thickness δ is therefore seen to be

$$\delta = \sqrt{\left\{\frac{2k\widetilde{\Delta T}x}{(a - 0{\cdot}5\tilde{a}) + \sqrt{[(a - 0{\cdot}5\tilde{a})^2 + 4k\widetilde{\Delta T}x(b - 0{\cdot}25\tilde{b})]}}\right\}}. \tag{A.6}$$

It remains to remove the product $\dot{m}\delta$ from the parameter P. Substituting (A.2) in (A.4), integrating the result, and extracting the coefficients a and b, there is obtained

$$\Gamma = (a\delta + b\delta^3)/(\lambda + 0{\cdot}354\,C_p\Delta T). \tag{A.7}$$

Now, $C_p\Delta T \ll \lambda$ for most applications; hence, $\lambda + 0{\cdot}354\,C_p\Delta T$ is seen to be a weak function of x. Thus, a comparison of (A.5) and (A.7) reveals that

$$\dot{m}\delta \cong k\Delta T/(\lambda + 0{\cdot}354\,C_p\Delta T) \tag{A.8}$$

and hence adequate forms of the coefficients a and b are:

$$a = (0{\cdot}5\,u_{v,e})\,(k\Delta T/\nu)/(1 + r) \tag{A.9}$$

and

$$b = (\rho g^*/3\nu)\,(\lambda + 0{\cdot}354\,C_p\Delta T)\,(4 + r)/(4 + 4r) \tag{A.10}$$

where

$$r = 1/(Pr)\,(\lambda/C_p\Delta T + 0{\cdot}354).$$

Since the local temperature distribution is linear, h^t is equal to k/δ where h^t denotes a theoretical heat transfer coefficient for the non-isothermal wall problem. If (A.6) is substituted for δ and the result is rearranged, h^t may be written as

$$h^t = \frac{1}{2}\sqrt{\left\{f_1\frac{k^2}{\nu x}\,\frac{u_{v,e} + \sqrt{[u_{v,e}^2 + (16g^*x/r)f_2]}}{2}\right\}} \tag{A.11}$$

where

$$f_1 = (k\Delta T)\,(2 - \tilde{a}/a)/(1 + r)\,(k\widetilde{\Delta T})$$

and

$$f_2 = \frac{k\widetilde{\Delta T}}{k\Delta T}\,\frac{(4 + r)\,(1 + r)}{4}\,\frac{1}{3}\,\frac{(4 - \tilde{b}/b)}{(2 - \tilde{a}/a)^2}.$$

For an isothermal wall, the functionals f_1 and f_2 reduce to

$$f_{1,iso} = 1/(1 + r) \simeq 1$$

and

$$f_{2,iso} = (4 + r\,(1 + r)/4 \simeq 1.$$

We now explore an alternative to the application of (A.11) directly. Clearly, the ratio h^t/h^t_{iso} may be written as

$$\frac{h^t}{h^t_{iso}} = \sqrt{\left\{\frac{f_1}{f_{1,iso}}\,\frac{1 + \sqrt{[1 + (16g^*x/ru_{v,e}^2)f_2]}}{1 + \sqrt{[1 + (16g^*x/ru_{v,e}^2)f_{2,iso}]}}\right\}}. \tag{A.12}$$

Finally, careful study of (A.12) suggests that to a good approximation it could as well be written as

$$\frac{h^t}{h^t_{iso}} \simeq \sqrt{\left\{\left(\frac{2\Delta T}{\widetilde{\Delta T}} - 1\right)\frac{1 + \sqrt{[1 + (16g^*x/ru_{v,e}^2)\,(\widetilde{\Delta T}/\Delta T)/(2 - \widetilde{\Delta T}/\Delta T)^2]}}{1 + \sqrt{[1 + (16g^*x/ru_{v,e}^2)]}}\right\}} \tag{A.13}$$

Résumé—Un programme général de calculateur a été élaboré qui résout la mise sous différences finies des équations de conservation du type couche limite pour la condensation par film laminaire. De plus, les solutions sous forme analytique basées sur les hypothèses de Nusselt ont été étendues pour tenir compte de l'effet d'une paroi de condenseur non isotherme. En faisant comme cela, on définit une température de référence sous la forme $T_r = T_w + \alpha(T_s - T_w)$ pour évaluer les propriétés du fluide variant localement, qui tient compte à la fois des effets des variations de propriétés du fluide et des erreurs secondaires introduites par les hypothèses de Nusselt. Dans le développement numérique aussi bien que dans l'analytique, on a tenu compte de la traînée de la vapeur au moyen d'une solution asymptotique de la couche limite de vapeur avec forte aspiration qui est valable pour de nombreuses circonstances de la technique.

Cet article évalue la validité du résultat généralisé de Nusselt en fonction des effets (1) de l'écoulement forcé de la vapeur, (2) d'une température variable de la paroi, et (3) des propriétés variables du fluide. Dix fluides ont été étudiés, parmi lesquels l'eau, tous ayant des nombres de Prandtl supérieurs à l'unité. On a trouvé, sauf pour des variations importantes de la température pariétale, qu'une valeur de α dépendant seulement de l'espèce du fluide conduisait à une différence de 2 pour cent entre les prévisions théoriques et les résultats numériques.

Zusammenfassung—Es wurde ein allgemeines Rechenprogramm entwickelt, welches für laminare Filmkondensation die Erhaltungssätze mit Grenzschichtvereinfachungen und mit Hilfe von Differenzenverfahren löst. Zusätzlich werden analytische Lösungen in geschlossener Form angegeben, die, basierend auf den Nusselt'schen Vereinfachungen, den Einfluss nicht-isothermer Kondensationsflächen berücksichtigen. Dabei wird eine Bezugstemperatur zur Berücksichtigung der örtlich veränderlichen Stoffwerte angegeben, $T_r = T_w + \alpha(T_s - T_w)$ die sowohl die veränderlichen Stoffwerte als auch kleinere Fehler, die aus den Nusselt'schen Vereinfachungen stammen, erfasst. In den numerischen und den analytischen Ableitungen, wird die Dampfreibung mit Hilfe einer vereinfachten Lösung der Dampfgrenzschicht mit starker Absaugung, die für die meisten praktischen Fälle zutrifft, berücksichtigt.

Diese Arbeit zeigt die Gültigkeit der erweiterten Nusselt'schen Ergebnisse in Abhängigkeit von (i) erzwungener Dampfströmung, (ii) veränderlicher Wandtemperatur und (iii) veränderlichen Stoffwerten. Zehn Flüssigkeiten einschliesslich Wasser werden untersucht, alle mit Pr-Zahlen grösser als eins. Es zeigte sich, ausser bei sehr starker Änderungen der Wandtemperatur, dass für einen Wert von α, der nur von den Eigenschaften der Flüssigkeit abhängt, die Abweichungen der analytischen Berechnungen von den numerischen Ergebnissen höchstens 2 Prozent betragen.

Аннотация—Разработана общая программа для решения конечно-разностных аналогов уравнений сохранения в пограничном слое при ламинарной пленочной конденсации. Кроме того, получены замкнутые аналитические решения, основанные на допущениях Нуссельта, учитывающие влияние неизотермичности стенки конденсатора. При этом исходная температура для расчета локально-переменных свойств жидкости определяется в виде $T_r = T_w + \alpha\ (T_s - T_w)$, где учитывается как влияние изменения свойств жидкости, так и незначительные ошибки за счет допущений Нуссельта. В численном и аналитическом решениия сопротивление пара учитывалось с помощью асимптотического решения парового пограничного слоя при интенсивном отсосе, применимого в большинстве практических случаев.

В данной работе оценивается справедливость обобщенного результата Нуссельта с учетом влияния (а) вынужденного течения воды, (б) переменной температуры стенки, (в) переменных свойств жидкости. Исследованы десять жидкостей, включая воду, характеризующиеся числом Прандтля больше единицы. Установлено, что за исключением очень сильных изменений температуры стенки значение a зависящее тольдо от вида жидкости, дает результаты, отличающиеся от аналитических решений только на 2 %.